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The Efficacy of Model-Based Volatility Forecasting: Empirical Evidence in Taiwan

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Abstract

The paper adopts several time series models to assess the forecasting efficiency of future realized volatility in Taiwan stock market. The paper finds that, for 1-day directional accuracy forecast performance, semiparametric fractional autoregressive model (SEMIFAR, Beran and Ocker, 2001) ranks highest with 78.52% hit accuracy, followed by multiplicative error model (MEM, Engle, 2002), and augmented GJR-GARCH model. For 1-day forecasting errors evaluated by root mean squared errors (RMSE), GJR-GARCH model augmented with high-low range volatility ranks the highest, followed by SEMIFAR and MEM model, both of which, however, outperform augmented GJR-GARCH by the measure of mean absolute value (MAE) and *p*-statistics (Blair, Poon and Taylor, 2001).

Keywords: Realized volatility, range volatility, semiparametric fractional autoregressive model, multiplicative error model, GJR-GARCH model, variance gamma garch model

JEL Classification Codes: C22, C52, C53, G15

1. Introduction

Return volatility has become the center of financial economists in asset pricing and risk management in recent decade. The construction methods of return volatility and its forecasting efficiency are widely studied by a large amount of literature. The recent availability of tick data has also spurred the application of so called high frequency data in volatility measures. A widely used proxy for ex post daily volatility is the realized volatility (RV) by Andersen, Bollerslev, Diebold and Labys (2001).

The RV or realized variance is also called the integrated variance which can be approximated by summing the intraday squared returns with adequate sampling frequency (Merton, 1980, Andersen and Bollerslev, 1998). For a continuous diffusion process with jumps, the quadratic variation equals the sum of integrated volatility and the squared jumps through time. By Andersen et al. (2001), Barndorff-Nielsen and Shephard (2002) and Andersen et al. (2003), the RV will converge to the integrated volatility in the absence of jumps when sampling frequency goes to infinity. In the presence of jumps, the RV still provide consistent estimates of quadratic variation for the jump diffusion process.

The precision of RV measures tends to increase with the sampling frequency. However, a high sampling frequency creates more interval returns which are easily affected by market microstructure noise like bid-ask bounce, measurement error, nonsynchronous prices, etc. Relevant studies on the optimal choice of sampling frequency and the methods to isolate microstructure noise from realized volatility are found in literature. The literature also documents various sampling frequencies ranging from 5- to 30-minute (Andersen and Bollersleve, 1997; Andersen et al., 2003, Giot and Laurent, 2007). As the sampling frequency is not the main interest of the paper, the 20-minute intraday returns are used to construct the realized variance series. By assuming the computed RV as a benchmark, the paper explores more about the forecasting efficiency across econometric models.

To better fit the constructed realized volatility series with some stylized facts like fat tails, long memory, and high level of skewness and kurtosis, various models to forecast RV are adopted in literature. Andersen et al. (2003) show that, by using the logarithmic daily RV of exchange rates, a simple long-memory Gaussian vector autoregression performs better than other econometric models. Corsi (2004) proposes another form of long-memory model, HAR-RV, to forecast USD/CHF data with admirably better forecasting performance than other standard models like GARCH or ARFIMA. Lane (2006) further uses a mixture multiplicative error model (MEM) to forecast RV with a good forecasting result.

For comparison purpose, the paper includes the range volatility as an explanatory variable in the conditional variance by GJR-GARCH. The academic analysis of price range, which is defined as the difference between the highest and lowest prices of assets over some time interval, can be dated back to Parkinson (1980) and Garman and Klass (1980) and has seen a rapidly expanding literature on range-based volatility in recent years. The empirical results show that variances measured by range estimators efficiently approximate the daily integrated variance and the performance evaluation of range-based estimators can be found in Shu and Zhang (2006). Survey references include Poon (2005), McAleer and Medeiros (2008) and Chou et al. (2008).

The paper aims to compare several long memory models with other standard econometric models in forecasting efficiency for RV in Taiwan stock market. Several measures are also adopted to evaluate the forecasting performance of the models examined. The paper is structured as follows. Section 2 describes the data. Section 3 makes a brief review of the adopted models in theory. Section 4 is the empirical results of out-of-sample volatility forecasts and evaluations across models. Section 5 is the conclusion.

2. Data

2.1. Construction of Volatility Series

The daily closing index of TAIEX and intraday minute-tick index are obtained from Taiwan Economic Journal (TEJ). The full sample range starts from 1999/4/1 to 2007/12/31[1]. The daily realized volatility (RV) is obtained by summing the intraday 20-minute interval returns including the overnight returns [2]. The in-sample data range starts from 1999/4/1 to 2003/12/31 with 1,217 observations on daily returns and realized volatilities. The out-of-sample period starts from 2004/1/2 to 2007/12/31 with a total 992 trading days. The range-based volatility is computed by the formula suggested by Parkinson (1980):

[1] The data starts from 1999/4/1 as the intraday 1-minute trading index is not available in the database before that time. TAIEX has extended the daily trading time from 12:00 to 13:30 since 2001/1/2.

[2] Given the presence of microstructure effects in TAIEX market, the sampling frequency of 20-minute intraday return is adopted instead of the usually adopted 5-minute frequency in literature.

$$\hat{\sigma}_t^2 = \frac{\ln^2(h_t / l_t)}{4 \ln 2} \tag{1}$$

$\hat{\sigma}_t$ represents the intraday high-low range volatility on day *t*. h_t and l_t denote the highest and lowest price levels observed during trading on day *t*. The descriptive statistics are listed in Table 1. Similar to the empirical results in literature, both volatilities are right-skewed and leptokurtic. After taking logarithms, however, both logarithmic volatility series appear to be Gaussian.

Table 1: Daily Realized Volatility Distributions

	Mean	**St. Dev.**	**Skewness**	**Kurtosis**	**Q(20)[a]**
Realized volatility	0.096	0.022	1.017	4.235	2150.1
Logarithmic RV	-2.360	0.219	0.363	3.079	2119.5
Range volatility	0.007	0.004	2.065	8.819	1324.0
Logarithmic range volatility	-5.073	0.494	0.396	3.284	1425.7

a. Ljung-Box test statistics for up to twentieth order serial correlation.

3. Model Specifications

3.1. RV-based Models

3.1.1. Multiplicative Error Models

The goal of the paper is to evaluate the forecasting performance of numerous econometric models based on the high frequency daily realized volatility. Return-based models, i.e, models based on the assumptions of return process, are also estimated for comparison purpose and will be discussed in later section. The first model is the multiplicative error models (MEM) suggested by Engle (2002) to model the RV series. As the multiplicative models allow non-negative series to be modeled, MEM(*p*,*q*) models can be formulated as follows:

$$RV_t = \mu_t \, \epsilon_t \tag{2}$$

$$\mu_t = w + \sum_{i=1}^{p} \alpha_i \, RV_{t-1} + \sum_{j=1}^{q} \beta_j \, \mu_{t-1} \tag{3}$$

where μ_t is the conditional mean of the RV based on all information up to time *t*, and the error term ϵ_t is assumed to be independent and identically distributed (*iid*) with a unit mean from a non-negative distribution like exponential or gamma distribution. By Engle and Gallo (2006) and Lanne (2006), the gamma distribution seems to be the best choice for RV. The parameters of the gamma distribution are assumed to be constrained as $\epsilon_t \sim Gamma(\gamma, \delta)$, where $\delta = 1/\gamma$ is to ensure the error term to have a unit mean. The estimation of MEM(*p*,*q*) model is based on the annualized RV series by $\sqrt{252 * RV}$ and the result is listed in Table 2. The results reveal that MEM11 is not superior to MEM22, but while in making iterated forecasts for 1-day ahead volatilities, the MEM22 yields frequent convergence failures and therefore, MEM11 is selected as the forecasting model in the paper.

Table 2: Estimation results of multiplicative error model [a]

MEM(*p*,*q*)	ω	α_1	α_2	β_1	β_2	γ	**Logl**	**AIC**	**SC**
MEM11	0.012*** (4.051)	0.207*** (11.292)		0.746*** (31.357)		7.931*** (29.296)	1290.81	-2.115	-2.098
MEM12	0.012*** (3.774)	0.225*** (8.591)		0.583*** (4.015)	0.143 (1.163)	7.939 (29.132)	1291.46	-2.114	-2.093
MEM21	0.009** (3.382)	0.235 (8.775)	-0.064* (1.854)	0.794*** (27.761)	(28.998)	7.944***	1291.88	-2.115	-2.094
MEM22	0.001 (1.074)	0.232*** (9.878)	-0.208*** (8.567)	1.522*** (13.664)	-0.551*** (5.783)	7.991*** (28.661)	1295.62	-2.119	-2.094

a. *t*-statistics are given in parenthesis. *, ** and *** represent the significance level with *p*-value of 0,1, 0.05 and 0.01. Logl is the log-likelihood value. AIC and SC are Akaike information criterion and Schwartz Bayesian criterion, respectively.

3.1.2 Semiparametric Fractional Autoregressive (SEMIFAR) Model

Numerous empirical studies support an existence of long memory in financial volatility series (Lobato and Savin, 1998; Ray and Tsay, 2000). Andersen et al. (2001b) and Martens and Zein (2004) suggested using fractional ARIMA (FARIMA) with an estimated memory parameter (d) around 0.4. Beran and Ocker (1999, 2001) and Beran et al. (1999) proposed the semiparametric fractional autoregressive (SEMIFAR) model by introducing a deterministic trend along to the FARIMA(p,d,0) model [3]

$$\phi(L)(1-L)^{\delta}[(1-L)^{m} y_t - g(i_t)] = \epsilon_t \tag{4}$$

where L is the backshift operator, $\phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$ is a polynomial with roots outside the unit circle, $g(i_t)$ is a smooth trend function on [0,1] with $i_t = t/T$ and ϵ_t is *iid* normally distributed with mean zero and variance σ_{ϵ}^2. Beran and Ocker (2001) fitted SEMIFAR models with daily volatility series of 19 stock market indexes and found a significant trend among developed countries under consideration [4]. For comparison purpose, the short memory models like ARMA model is also estimated in forecasting future volatility. The model of ARMA(1,1) and AR(3) are adopted to forecast 1-, 5- and 22-day ahead volatilities.

3.2. Return-Based Models

3.2.1. GJR-GARCH Model

As the financial volatility is a latent factor and unobservable, models based on asset returns to imply out the dynamics of volatility become more feasible in most cases. This section starts with the widely-used generalized autoregressive heteroskedasticity model, or GJR-GARCH model developed by Glosten et al. (1993), to account for the effect of good news and bad news on conditional volatility. The daily trading volume and high-low range volatility are also considered separately as additional regressors in GARCH equation for further examination. The augmented GJR-GARCH model is denoted as the following:

$$R_t = \mu + \epsilon_t \tag{5}$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 s_{t-1}\epsilon_{t-1}^2 + \beta h_{t-1} + \delta W_{t-1} \tag{6}$$

where R_t: return on day t, h_t: conditional volatility on day t, s_{t-1}: 1 if $\epsilon_{t-1} < 0$ and 0 otherwise, W_{t-1}: intraday high-low range variance or return of trading volume on day *t-1*. s_t is an indicator function to account for effect of the good news ($\epsilon_t > 0$) and bad news ($\epsilon_t < 0$) on conditional variance. Therefore, the effect of good news is measured by α_1, and the effect of bad news is measured by $(\alpha_1 + \alpha_2)$. The

[3] The specifications and the estimation of SEMIFAR can be referenced to Zivot and Wang (2006), chapter 8, p293-295.
[4] The volatility series used is the power-transformed absolute difference, i.e, $y_t = |I_t - I_{t-1}|^{0.25}$, where I_t is the closing index.

effects of high-low range volatility or trading volume on conditional variance are measured by coefficient δ.

3.2.2. Fractionally Integrated GARCH (FIGARCH) and EGARCH (FIEGARCH) Model

In view of the high persistence in GARCH models, Baillie et al. (1996) further introduced ARMA terms into GARCH model in terms of squared residuals. For the GARCH(*p, q*) model:

$$h_t = a + \sum_{i=1}^{p} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} b_j h_{t-1} \tag{7}$$

where ϵ_t^2 can be expressed as follows:

$$\phi(L)\epsilon_t^2 = a + b(L)u_t \tag{8}$$

and:

$$u_t = \epsilon_t^2 - h_t$$

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_m L^m$$

$$b(L) = 1 - b_1 L - b_2 L^2 - \cdots - b_q L^q$$

m=max(p, q) and $\phi_i = a_i + b_i$. By extending the ARMA(m, q) in Equation (8) based on the condition of $0<d<1$, the specifications of the conditional variance can be written as:

$$b(L)\, h_t = a + [b(L) - \phi(L)(1-L)^d]\epsilon_t^2 \tag{9}$$

Equation (9) [5] is the fractionally integrated GARCH or FIGARCH(*m*, *d*, *q*) called by Baillie et al. (1996). Bollerslev and Mikkelsen (1996) further propose FIEGARCH model as the following:

$$\phi(L)(1-L)^d \ln h_t = a + \sum_{j=1}^{q}(b_j \,|\, x_{t-j} \,| + \gamma_j x_{t-j}) \tag{10}$$

where x_t is the standardized residual $\epsilon_t / \sqrt{h_t}$ and $\gamma_j \neq 0$ allows the leverage effects. The estimation results are listed in Table 3.

Table 3: Estimation Results of FIGARCH and FIEGARCH Model [a]

	c	a	a_1	a_2	b_1	γ_1	γ_2	d	**Logl**	**AIC**	**SC**
FIEGARCH11	0.043 (0.937)	0.198** (3.263)	-0.089 (0.103)		0.259** (0.142)			0.366*** (0.071)	-2377	4763	4789
FIEGARCH21	0.022 (0.491)	-0.156*** (-4.845)	-0.009 (-0.190)	0.219*** (3.644)	0.638*** (3.784)	-0.112*** (-3.330)	0.031 (0.793)	0.431*** (3.613)	-2361	4738	4779

a. *t*-statistics are given in parenthesis. *, ** and *** represent the significance level with *p*-values of 0,1, 0.05 and 0.01.
Logl is the log-likelihood value. AIC and SC are Akaike information criterion and Schwartz Bayesian criterion, respectively.

The selection of the lag terms for ARCH and GARCH in FIGARCH and FIEGARCH are mainly based on the Akaike information or Schwartz Bayesian criterion (AIC/SC). The estimated fraction difference parameter (d) for both models falls in the range (0, 0.5), suggesting the existence of long memory for index returns. The percentage returns are used in estimating FIGARCH and FIEGARCH as the daily returns are very small numbers which may cause the failure of convergence.

3.2.3 Variance Gamma GARCH Model

The variance gamma process has been popular in modeling jumps of asset returns as an alternative to Black-Scholes option pricing model (Madan et al., 1998; Carr and Wu, 2004). To better capture the stylized facts of asset returns with a fat-tailed distribution and volatilities which are clustered and long-range dependent, the logarithmic asset returns under physical measure $\mathbb{P}$ are assumed to be GARCH(1,1)-VG(0, σ, ν) (henceforth VGGARCH) as in the following:

[5] The FIGARCH and FIEGARCH equations and parameters estimations can be referenced to Zivot and Wang (2006).

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{1}{2} g_t + \lambda\sqrt{h_t} + \epsilon_t$$
$$\epsilon_t \mid \Omega_{t-1} \sim \mathrm{VG}(0, h_t, \nu)$$
$$h_t = w + a\,\epsilon_{t-1}^2 + b\,h_{t-1}$$

where $g_t = (-2/h_t)\log(1 - h_t\nu/2)$ and ϵ_t is a sequence of independent random variables with mean 0 and variance h_t. The risk-free interest rate r is assumed to be constant in every period. The larger jumps appear more frequently and cluster more together as ν gets bigger, which creates greater volatility in the GARCH structure. λ is the market price of risk.

4. Empirical Results

4.1. Forecasting Evaluations

The models adopted by the paper to assess the forecasting efficiency of future realized volatility are classified into two groups: six return-based models and four RV-based models. The first three return-based models include GJR and its augmented ones, i.e, GJRhl and GJRvol, which denote the additional regressors of high-low range volatility and trading volume in conditional variance equation of GARCH, respectively. The fourth model is VGGARCH model. The fifth and the sixth are FIGARCH and FIEGARCH models. For RV-based models, SEMIFAR, MEM(1,1), ARMA(1,1) and AR(3) are considered. After initial parameter estimations, the out-of-sample forecasts for 1-, 5- and 22-day ahead volatilities are obtained from the estimated models. The 5-day forecast represents 1-week ahead forecast. The 22-day forecast, an assumed number of trading days for a calendar month, represents 1-month ahead forecast. 5- and 22-day ahead volatility forecasts are obtained by summing the daily forecasted volatility with fixed estimated parameters in the models. The procedure is reiterated on a moving window basis. The out-of-sample period thus yields 992 one-day forecasts, 198 five-day forecasts and 45 twenty two-day forecasts.

Figure 1: RV vs. forecasted volatility series from 2004/1/2-2007/12/31

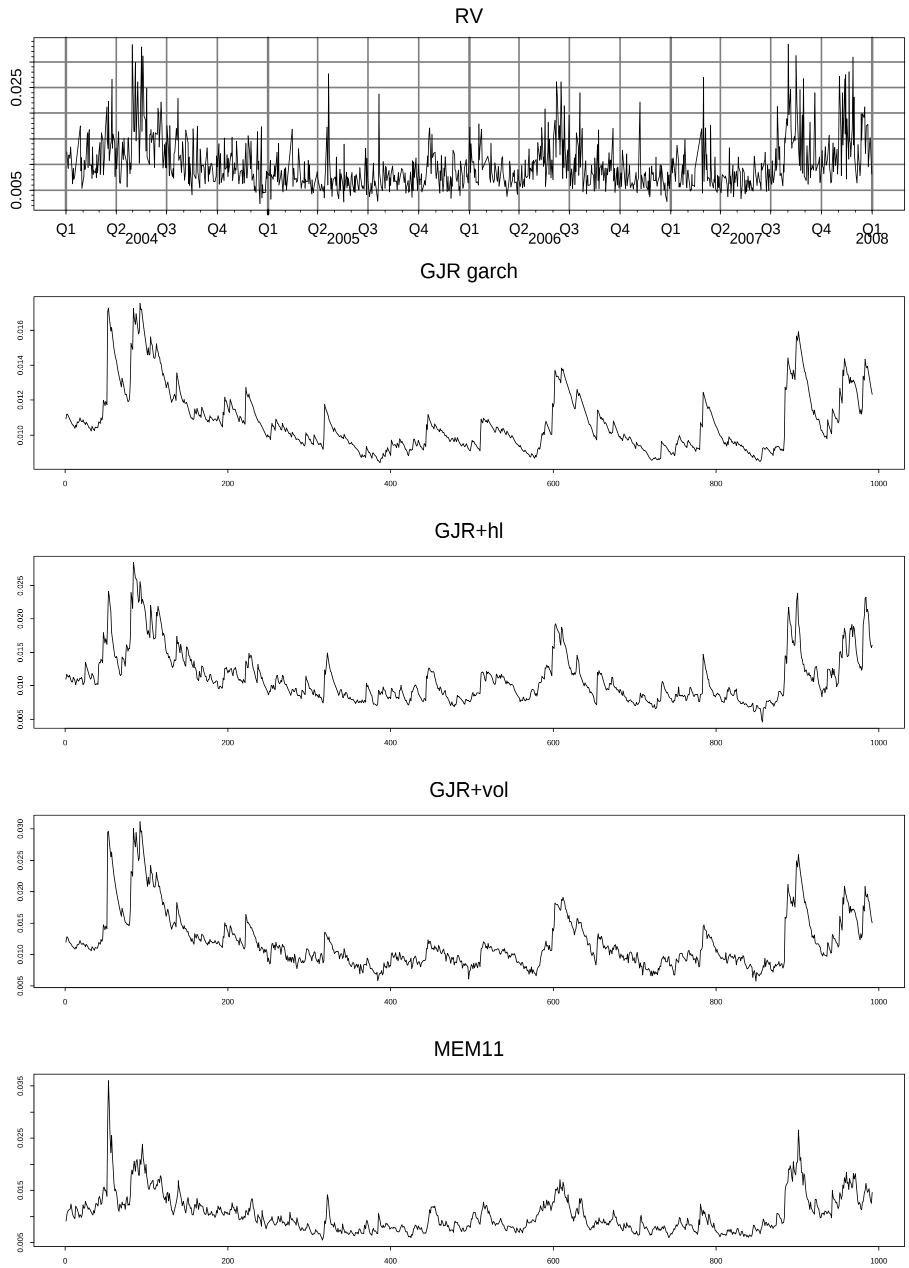

Figure 2: RV vs. forecasted volatility series from 2004/1/2-2007/12/31

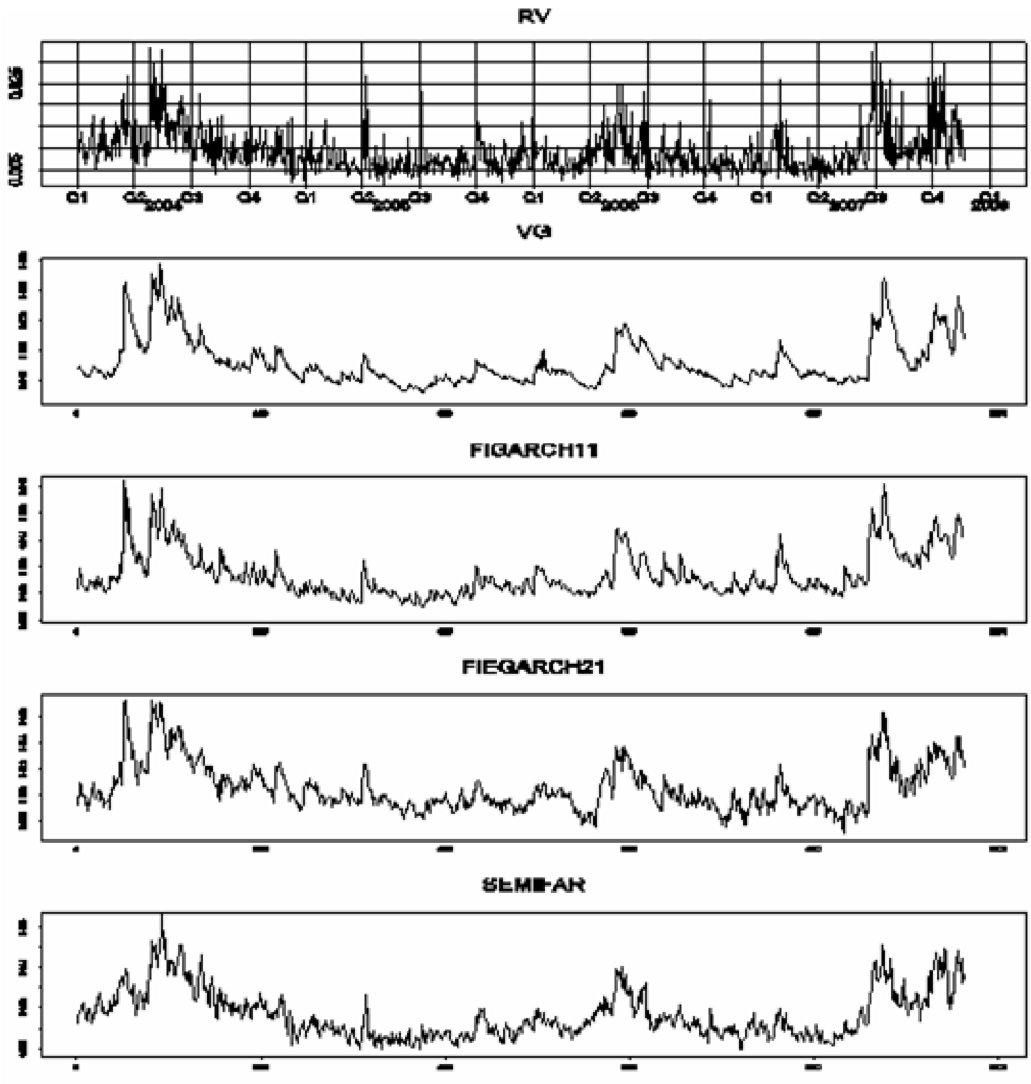

Figure 1 and 2 are 1-day ahead volatility forecasts by return-based models and RV-based models. The ex-post daily realized volatility is also placed for comparison purpose. By visual inspections, forecasted volatility series generated by all models seem to reveal spikes and troughs at positions similar to the RV series. However, forecasted series generated by FIEGARCH, SEMIFAR show more zigzags or frequent oscillations found in RV series than other models. Smooth slopes or flat line segments are found in series generated by GJR family and VGGARCH model.

The forecast performance of various models is summarized in Table 4. Four measures are used to evaluate the model and are listed in the following:

- Hit percentage: This is the percentage of correct directions of movements for 1-day ahead forecasts. By Dahl and Hylleberg (1999), a 2×2 contingency table which summarizes the number of correct forecasted ups and downs is listed as in the following:

		Predicted		
		up	**down**	**Subtotal**
Actual	up	n_{uu}	n_{ud}	n_{u0}
	down	n_{du}	n_{dd}	n_{d0}
	subtotal	n_{0u}	n_{0d}	n

where n is the total number of 1-day ahead forecasts, n_{uu} is the number of days when both the actual outcome and the predicted are up, n_{dd} is the number of days when both the actual and the predicted are down. The forecasted values can be evaluated by the following test statistic:

$$\chi^2 = \sum_{i\in\{u,d\}} \sum_{j\in\{u,d\}} \frac{(n_{ij} - n_{i0}n_{0j}/n)^2}{n_{i0}n_{0j}/n} \sim \chi^2(1) \tag{11}$$

A small p value rejects the null hypothesis that the model does not outperform the chance of random choice.

- P-statistic: Suggested by Blair et al. (2001), P-statistic measures the proportion of the variances of realized volatilities explained by volatility forecasts.

$$P = 1 - \frac{\sum_{i=1}^{(T-S)/N} (y_{S+N(i-1),N} - \hat{y}_{S+N(i-1),N})^2}{\sum_{i=1}^{(T-S)/N} (y_{S+N(i-1),N} - \overline{y})^2} \tag{12}$$

$y_{t,N}$ denotes realized volatility measured as the sum of daily RV over an N-day forecast period beginning on day t, and $\hat{y}_{t,N}$ denotes the corresponding volatility forecast for the same forecast period. For the data of TAIEX used in the paper, T=2,209, all observations of the whole sample, and S=1,217, the number of in-sample observations. N is the forecast horizon of 1, 5 and 22 days.

- Root mean square error (RMSE):

$$\text{RMSE} = \frac{1}{(T-S)/N} \sum_{i=1}^{(T-S)/N} \left(y_{S+N(i-1),N} - \hat{y}_{S+N(i-1),N}\right)^2 \tag{13}$$

- Mean absolute error (MAE):

$$\text{MAE} = \frac{1}{(T-S)/N} \sum_{i=1}^{(T-S)/N} | y_{S+N(i-1),N} - \hat{y}_{S+N(i-1),N} | \tag{14}$$

Table 4: Statistics of Out-of-Sample Volatility Forecast Using Realized Volatility as a Benchmark across Models for TAIEX Index [a]

Models	1-day Forecast				5-day Forecast			22-day Forecast		
	Hit	RMSE	MAE	*P*-value	RMSE	MAE	*P*-value	RMSE	MAE	*P*-value
GJR	0.6683	0.00415	0.0033	0.2378	0.0153	0.0149	-0.1191	0.0721	0.0739	-0.7993
GJRhl	0.7016	0.00401	0.0032	0.2527	0.0099	0.0073	0.6845	0.0262	0.0205	0.8089
GJRvol	0.6421	0.00436	0.0037	0.0224	0.0152	0.0154	-0.1667	0.0752	0.0816	-1.0882
VGGARCH	0.6452	0.00436	0.0036	0.0606	0.0163	0.0146	-0.0649	0.0761	0.0647	-0.6003
FIGARCH	0.6683	0.00416	0.0032	0.2463	0.0140	0.0128	0.1806	0.0632	0.0621	-0.2367
FIEGARCH	0.6743	0.00412	0.0031	0.2708	0.0147	0.0129	0.1487	0.0873	0.0846	-1.7569
SEMIFAR	0.7852	0.00406	0.0028	0.2825	0.0129	0.0092	0.4598	0.0507	0.0344	0.4064
MEM11	0.7056	0.00416	0.0030	0.2577	0.0126	0.0095	0.4986	0.0495	0.0365	0.4465
ARMA11	0.6603	0.00406	0.0033	0.2242	0.0131	0.0126	0.2662	0.0556	0.0638	-0.2051
AR(3)	0.6592	0.00422	0.0038	0.0526	0.0152	0.0180	-0.3497	0.0723	0.0983	-1.7643

a. Hit is the percentage of directional accuracy for 1-day forecast. The other measures of forecast accuracy are root mean square error (RMSE), mean absolute error (MAE) and P-Statistics which is suggested by Blair et al. (2001).

For the 1-day ahead directions forecasts in Table 4, SEMIFAR ranks the highest hit percentage, followed by MEM11 and GJRhl model, which all three models have P values less than 0.01 and thus reject the null hypothesis of no superiority over random walks. As for the performance of 1-day forecasting errors in terms of RMSE, GJRhl outperforms all other models. SEMIFAR and MEM11 ranks the second and the third. However, they outperform GJRhl in terms of MAE and P-statistic. FIEGARCH has the highest P-statistic but lacks consistency for the criterions of RMSE and MAE. Therefore, SEMIFAR model seems to have better performance in terms of accuracy of direction forecasting and consistently outperforms GJRhl and MEM11 models based on error criterions. For 5- and 22-day volatility forecasts, however, GJRhl consistently outperforms SEMIFAR and MEM11 models in terms of three error criterions.

4.2 Econometric Analysis

The regressions of RV on the forecasted series from previous models are conducted in a non-overlapping framework (Christensen and Prabhala, 1998):

$$\ln(y_{t,N}) = \alpha + \beta \ln(\hat{y}_{t,N}) + e_t \qquad (15)$$

where $y_{t,N}$ denotes realized volatility measured as the sum of daily RV over an *N*-day forecast period beginning on day *t*. $\hat{y}_{t,N}$ is the forecasted volatility series generated from models discussed in previous section. The use of log-transformed volatility series is recommended by Andersen et al. (2001a) to make the distribution of the error term close to normal density. Regressions based on the transformed-log series are also found to easily accept the null hypothesis of no heteroscedasticity in error terms. Due to limited space, only α, β and adjusted R^2 estimated by OLS using Newey-West standard errors are reported in Table 5.

Table 5: Regression of Out-of-Sample Volatility Forecasts across Models on Log Realized Volatilities [a]

Models	1-day forecast			5-day forecast			22-day forecast		
	α	β	R^2	α	β	R^2	α	β	R^2
GJR	-1.2639 (-7.2869)	0.7723 (19.9750)	0.2865	-0.9519 (-5.4778)	0.7412 (12.2340)	0.4301	-0.8356 (-4.4655)	0.5443 (4.0006)	0.2543
GJRhl	-1.0051 (-5.7214)	0.8175 (21.1970)	0.3115	-0.7240 (-7.6071)	0.7679 (24.7980)	0.7571	-0.2768 (-4.4216)	0.8482 (21.0900)	0.9098
GJRvol	-1.4891 (-8.7047)	0.7205 (18.9360)	0.2652	-0.8965 (-5.5274)	0.7638 (13.4540)	0.4775	-0.8896 (-4.5525)	0.5153 (3.5472)	0.2084
VGGARCH	-1.3929 (-7.7404)	0.7394 (18.5330)	0.2568	-1.1200 (-6.2130)	0.6722 (10.8570)	0.3724	-0.8428 (-4.8816)	0.5111 (4.3039)	0.2848
FIGARCH	2.7348 (7.5978)	1.6346 (20.7240)	0.3019	-0.6612 (-3.7106)	0.8303 (13.5590)	0.4814	-0.6641 (-3.2156)	0.6489 (4.4511)	0.2995
FIEGARCH	1.8769 (5.6979)	1.4417 (20.0420)	0.2879	-0.9528 (-5.6747)	0.7244 (12.6590)	0.4470	-0.9238 (-6.2378)	0.4919 (4.4909)	0.3034
SEMIFAR	-0.0009 (-0.0040)	0.9982 (21.2020)	0.3116	0.8175 (3.0603)	1.2625 (14.5680)	0.5174	0.6865 (2.0291)	1.4289 (6.6956)	0.4990
MEM11	-0.9753 (-5.5475)	0.8109 (21.3500)	0.3146	-0.3075 (-1.5577)	0.9151 (14.0300)	0.4985	-0.2367 (-1.1406)	0.8687 (6.4949)	0.4835
ARMA11	-0.0845 (-0.3742)	1.0264 (20.5520)	0.2984	-0.1314 (-0.5733)	1.0175 (12.8430)	0.4542	-0.3156 (-1.1540)	0.9315 (4.6249)	0.3167
AR(3)	-0.0538 (-0.2034)	1.0501 (17.6530)	0.2387	-0.2200 (-0.6423)	1.0270 (8.3261)	0.2575	-1.2424 (-3.0242)	0.2705 (0.8017)	-0.0082

a. The parenthesis below each estimated coefficient is the t-statistic based on Newey-West standard errors.

The null hypothesis of heteroscedasticity is accepted at p=0.05 only for AR(3) for 5- and 22-day ahead forecast. The hypothesis of unbiased forecast is further tested on the estimated coefficients with H_0: $\alpha = 0$ and $\beta = 1$. Wald tests conducted on the coefficients across models show that only SEMIFAR 1-day forecast and MEM11 22-day forecast are unbiased forecast for RV series. To mitigate the potential bias from large sample size leading to Type II errors, the critical test statistic values of t and F are also adjusted[6]. GJRhl model is found to deliver the highest adjusted R^2 values for 5- and 22-day ahead forecast, which is consistent with the best outcome by three measures for GJRhl.

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[6] As shown by Lindley's paradox, the *t*-and *F*-statistics are adjusted when their absolute values are greater than the value of the following:

$$t^* = \sqrt{T-k} \times (T^{1/T} - 1), F^* = \frac{T-1}{K-1} \times (T^{(k-1)/T} - 1)$$

where *T* is the sample size and *k* is the number of degrees of freedom lost in the regression.

5. Conclusion

The realized volatility computed from intraday data has been widely regarded as a more accurate proxy for market volatility than squared daily returns. To further evaluate the efficacy of the high-low range volatility, which is a measure with reviving studies recently, GJR-GARCH includes the range volatility as an additional explanatory variable to forecast the conditional variance. Various models based on the realized volatility are presented in the paper and compared for their forecasting efficiency.

Several measures are adopted to evaluate the forecasting efficiency. The most naïve measure is the 1-day ahead directional forecast. The long-memory model of SEMIFAR performs best with a 78% hits among all models examined. MEM and GJRhl model rank the second and the third with negligible difference in hit percentage. However, GJRhl performs best by the measure of RMSE. For 5- and 22-day ahead forecast, GJRhl consistently outperforms SEMIFAR and MEM11 by the measure of RMSE, MAE and *P*-statistics.

The empirical results confirm that the long-memory model, SEMIFAR, is able to deliver good quality of directional forecasts and merits further studies on its application in volatility trading and risk management. Additionally, the high-low range volatility, when combined with GJR-GARCH model, is able to deliver good quality of forecasted volatility values. GJRhl model also has the highest predictive ability in the framework of forecasting regressions. The strong empirical evidence supports a possible reconciliation between GJRhl and SEMIFAR model to obtain a more robust econometric model in forecasting volatility.

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